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Quantization of the sum of products of consecutive reciprocals

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The classical formula: $\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{(n+1)}$ is quantized.

Theorem 1. For $n \in \mathbb{Z}_{\geq 1}$,

$$\sum_{k=1}^{n} \frac{1}{[k]_q^{\sim}[k+1]_q^{\sim}} = q - \frac{q^{n+1}}{[n+1]_q^{\sim}}, \tag{1}$$

where

$$[x]_q^{\sim} = \frac{q^x - q^{-x}}{q - q^{-1}},$$

is the second quantization of x.

Proof. The formula follows at once, if we take an obvious formula

$$[n+1]_q^{\sim} = q[n]_q^{\sim} + q^{-n}, \tag{2}$$

and divide it by $\frac{q^{-n}}{[n]_q^{\sim}[n+1]_q^{\sim}}$, resulting in the telescopic formula

$$\frac{1}{[n]_q^{\sim}[n+1]_q^{\sim}} = \frac{q}{[n]_q^{\sim}} - \frac{q^{n+1}}{[n+1]_q^{\sim}}. \tag{3}$$

Summing it up on n, we get (1), because

$$\frac{q^1}{[1]_q^{\sim}} = q. \quad \blacksquare$$