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A quantum cubic representation for 16

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The identity $16 = 2{\cdot}2{\cdot}4$ is given a quantum cubic representation.

***Theorem* 1.** For $n \in \mathbb{Z}_{\geq 3}$, we have

$$\left([2]^{\sim}_{q}\right)^2 [4]^{\sim}_{q^n} = [3n+3]^{\sim}_{q} + [3n+1]^{\sim}_{q} + [n+3]^{\sim}_{q} + [n+1]^{\sim}_{q} - \left([3n-3]^{\sim}_{q} + [3n-1]^{\sim}_{q} + [n-3]^{\sim}_{q} + [n-1]^{\sim}_{q}\right). \quad (1)$$

where

$$[x]^{\sim}_{q} = \frac{q^x - q^{-x}}{q - q^{-1}}$$

is the symmetric 2[nd] quantization of x.

Proof. We will use the following formula (see Morton 2009):

$$[m+a]^{\sim}_{q} - [m-a]^{\sim}_{q} = [2]^{\sim}_{q^m} [a]^{\sim}_{q} . \quad (2)$$

With this, the right side of (1) becomes:

$$[2]^{\sim}_{q^{3n}}\left([3]^{\sim}_{q} + 1\right) + [2]^{\sim}_{q^n}\left([3]^{\sim}_{q} + 1\right),$$

and because

$$[3]^{\sim}_{q} + 1 = \left([2]^{\sim}_{q}\right)^2,$$

(1) reduces to

$$[4]^{\sim}_{q^n} \stackrel{?}{=} [2]^{\sim}_{q^{3n}} + [2]^{\sim}_{q^n} ,$$

or

$$[4]^{\sim}_{q} \stackrel{?}{=} [2]^{\sim}_{q^3} + [2]^{\sim}_{q} ,$$

which is obvious, because

$$[2]^{\sim}_{q^3} + [2]^{\sim}_{q} = (q^3 + q^{-3}) + (q + q^{-1}) = [4]^{\sim}_{q} \qquad \blacksquare$$

References

Morton, T. S., "A Product of two quantum integers," *Journal of Scientific and Mathematical Research* **3** (2009) pp. 5-6.