J. Sci. Math. Res. (2010) Vol. 4, p. 15 Published by Centaurus Press
(Received 2009-05-18)

Number 12 in the second quantization

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We give a quantum cubic representation of $12 = 2 \cdot 2 \cdot 3$.

***Theorem* 1.** For every $n \in \mathbb{Z}_{\geq 4}$, we have:

$$\left([2]_q^{\sim}\right)^2 [3]_{q^n}^{\sim} = [1]_q^{\sim} + [3]_q^{\sim} + [2n+3]_q^{\sim} + [2n+1]_q^{\sim} - \left([2n-1]_q^{\sim} + [2n-3]_q^{\sim}\right), \tag{2}$$

where

$$[x]_q^{\sim} = \frac{q^x - q^{-x}}{q - q^{-1}}$$

is the 2nd quantization of x.

Proof. Since

$$1 + [3]_{q^n}^{\sim} = 1 + q^2 + 1 + q^{-2} = (q^1 + q^{-1})^2 = ([2]_q^{\sim})^2,$$

our (2) becomes:

$$([2]_q^{\sim})^2 \left([3]_{q^n}^{\sim} - 1\right) \overset{?}{=} [2n+1]_q^{\sim} + [2n+3]_q^{\sim} - \left([2n-1]_q^{\sim} + [2n-3]_q^{\sim}\right). \tag{3}$$

The right side of (3) is:

$$\frac{1}{q - q^{-1}}\left\{q^{2n+1} - q^{-2n-1} + q^{2n+3} - q^{-2n-3} - q^{2n-1} + q^{1-2n} - q^{2n-3} + q^{3-2n}\right\}$$

$$= \frac{1}{q - q^{-1}}\left\{q^{2n+1}(1 - q^{-4}) + q^{3-2n}(1 - q^{-4}) + q^{2n+3}(1 - q^{-4}) + q^{1-2n}(1 - q^{-4})\right\}$$

$$= \frac{q^{-2}(q^2 - q^{-2})}{q - q^{-1}}\left\{q^{2n+1} + q^{3-2n} + q^{2n+3} + q^{1-2n}\right\}$$

$$= [2]_q^{\sim}\left\{q^{2n-1} + q^{1-2n} + q^{2n+1} + q^{-1-2n}\right\} = [2]_q^{\sim}\left\{q^{2n-1}(1 + q^2) + q^{-1-2n}(1 + q^2)\right\}$$

$$= [2]_q^{\sim}\left\{q^{2n-1}q(q^{-1} + q^1) + q^{-1-2n}q(q^{-1} + q^1)\right\}$$

$$= [2]_q^{\sim}[2]_q^{\sim}\left\{q^{2n} + q^{-2n}\right\} = [2]_q^{\sim}[2]_q^{\sim}[3]_q^{\sim}$$

which is the left side of (3). ■