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# Quantum exponential function

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We quantize the usual exponential function.

The function

$$E(ax) = \exp(ax) = \sum_{n=0}^{\infty} \frac{a^n x^n}{n!} \tag{1}$$

satisfies

$$\frac{\mathrm{d}}{\mathrm{d}x} E(ax) = aE(ax) \,. \tag{2}$$

Replacing $\frac{\mathrm{d}}{\mathrm{d}x}$ by its symmetrical quantum counterpart $\frac{\mathrm{d}}{\mathrm{d}_q^{\sim} x}$:

$$\frac{\mathrm{d}f}{\mathrm{d}_q^{\sim} x} = \frac{f(qx) - f(q^{-1}x)}{q - q^{-1}} \,, \qquad x \neq 0 \,, \tag{3}$$

so that

$$\frac{\mathrm{d}}{\mathrm{d}_q^{\sim} x} (x^s) = [s]_q^{\sim} x^{s-1} \,, \qquad s \in \mathbb{R} \ (\text{or } \mathbb{C}),$$

$$[s]_q^{\sim} = \frac{q^s - q^{-s}}{q - q^{-1}} \,,$$

the corresponding $q$-version of $e^{ax}$ becomes:

$$E(ax, q) = \sum_{n=0}^{\infty} \frac{a^n x^n}{[n]_q^{\sim}!} \,.$$

Here,

$$[k]_q^{\sim}! = [1]_q^{\sim} \ldots [k]_q^{\sim} \,, \qquad k \in \mathbb{Z}_{\geq 1} \,, \ [0]_q^{\sim}! = 1 \,.$$

Indeed,

$$\frac{\mathrm{d}f}{\mathrm{d}_q^{\sim} x} [E(ax)] = \sum_{n=0}^{\infty} \frac{a^n x^{n-1}}{[n-1]!} = aE(ax, q) \,,$$

the desired $q$-analog of the classical relation (2).