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Cubic two in the second quantization

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We quantize 8 as the cube of 2 and decompose it into a sum of 4 quantum integers.

We decompose $\left([2]_q^\sim\right)^2 [2]_{q^n}^\sim$ into a sum of 4 quantum integers.

***Theorem* 1.** For $n \in \mathbb{Z}_{\geq 3}$, we have:

$$\left([2]_q^\sim\right)^2 [2]_{q^n}^\sim = [n+3]_q^\sim + [n+1]_q^\sim - \left([n-3]_q^\sim + [n-1]_q^\sim\right), \tag{1}$$

where

$$[x]_q^\sim = \frac{q^x - q^{-x}}{q - q^{-1}}$$

is the 2$^{\text{nd}}$ (symmetric under exchange $q \to q^{-1}$) quantization.

Proof. Using the obvious formula

$$[A]_q^\sim + [B]_q^\sim = [2]_{q^{(A-B)/2}}^\sim \left[\frac{A+B}{2}\right]_q^\sim, \tag{2}$$

which is a special case of the more general relation found previously (Kupershmidt 2009), we have:

$$[n+a]_q^\sim - [n-a]_q^\sim = [n+a]_q^\sim + [a-n]_q^\sim = [2]_{q^n}^\sim [a]_q^\sim$$

For $a = 3, 1$, we add up these formulae as in (1), and for the RHS of (1) get:

$$\text{RHS} = [2]_{q^n}^\sim \left\{[3]_q^\sim + 1\right\} = [2]_{q^n}^\sim \left\{q^2 + 1 + q^{-2} + 1\right\} = [2]_{q^n}^\sim \left([2]_q^\sim\right)^2$$

which is the LHS. ■

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References

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